Wavelets Applied to the Detection of Point Sources of UHECRs

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Scientific Motivations

- Detection of point sources of UHECRs is one of the main goals of PAO
- The signal of the possible sources are embedded in a stochastic (or unknown pattern of) background
- Wavelet filters have shown excellent results on the detection of faint point sources in cosmic microwave background (CMB)
- The Mexican Hat Wavelet Family was used to amplify the signal-to-noise ratio in CMB maps and enhance the detection of sources ("foreground" => noise) which could contaminate the CMB radiation (signal) maps
- In UHECRs research, the problem is the opposite: one wants to detect the sources (signal) and exclude the background (noise)

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The Mexican Hat Wavelet Family. Application to point source detection in CMB maps

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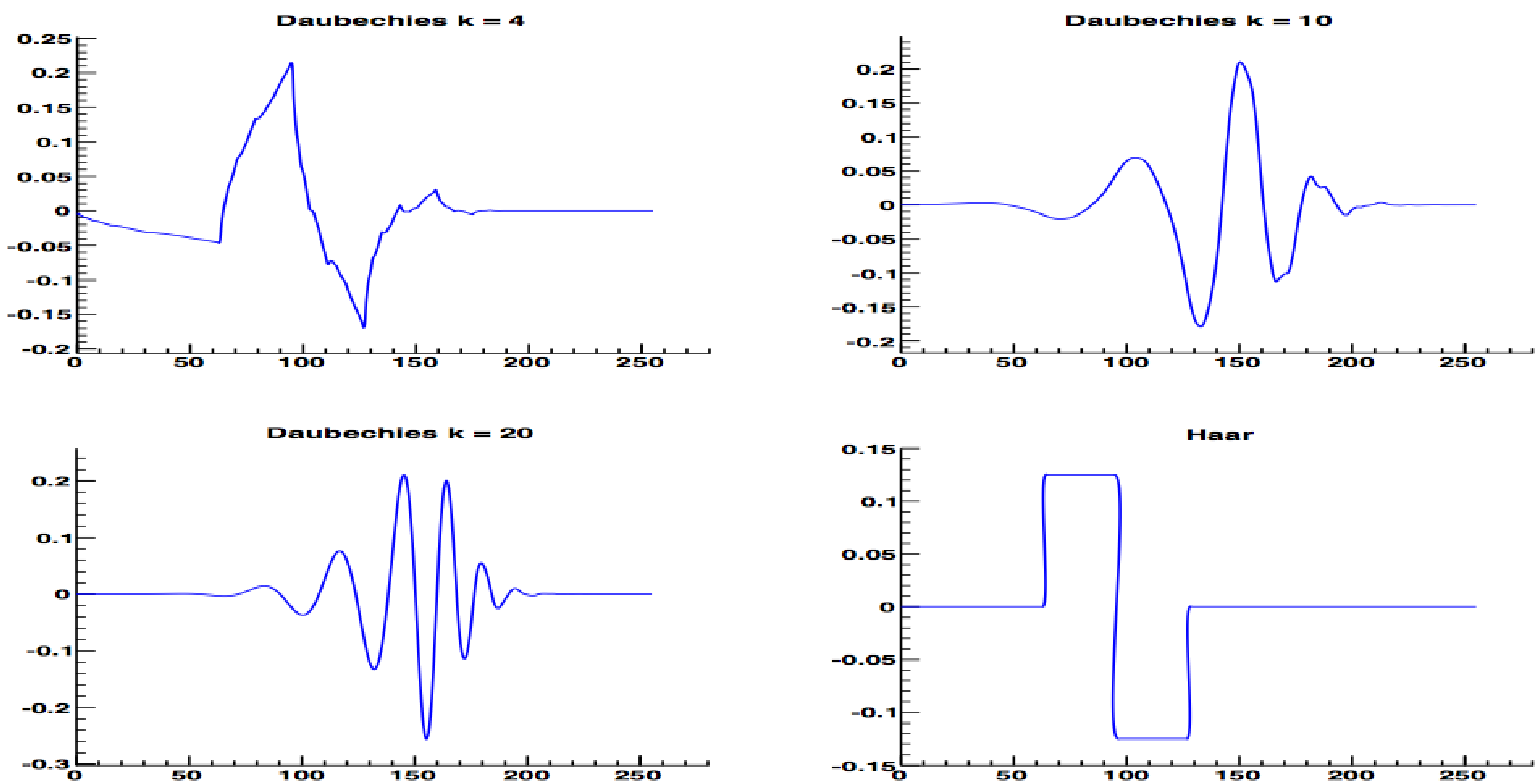
Wavelets

- Wavelets are localized wave-like oscillating functions belonging to the L^2 space
- These figures show some example of wavelets in 1-dimension

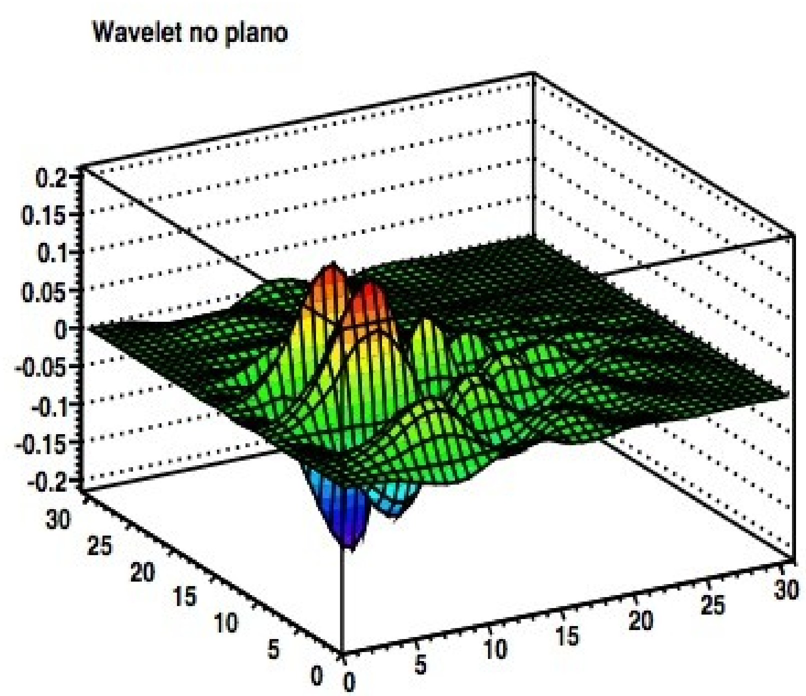


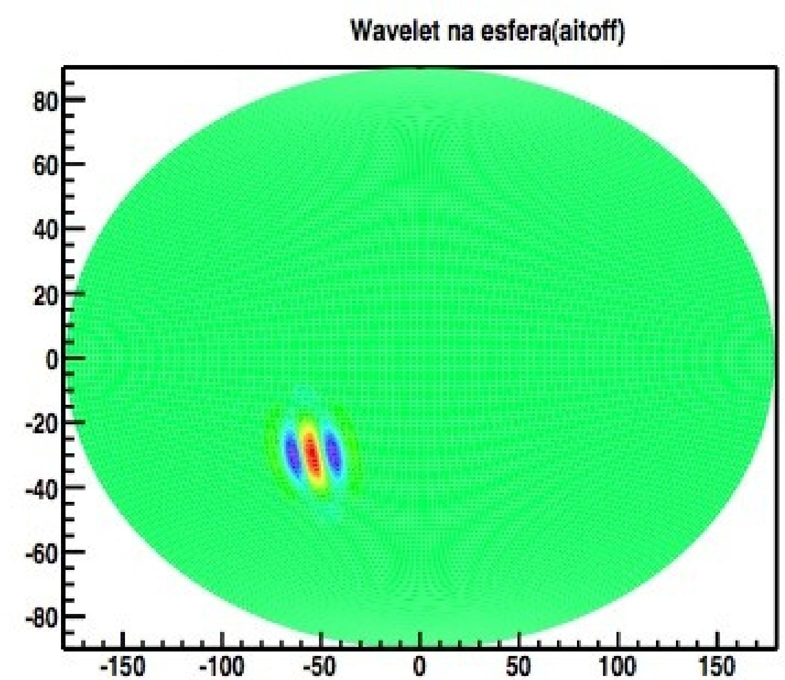
Wavelets

- When are wavelets useful?

"... if we are interested in transient phenomena - a word pronounced at a particular time, an apple located in the left corner of an image - the Fourier transform becomes a cumbersome tool."
(Mallat, Wavelet Tour)

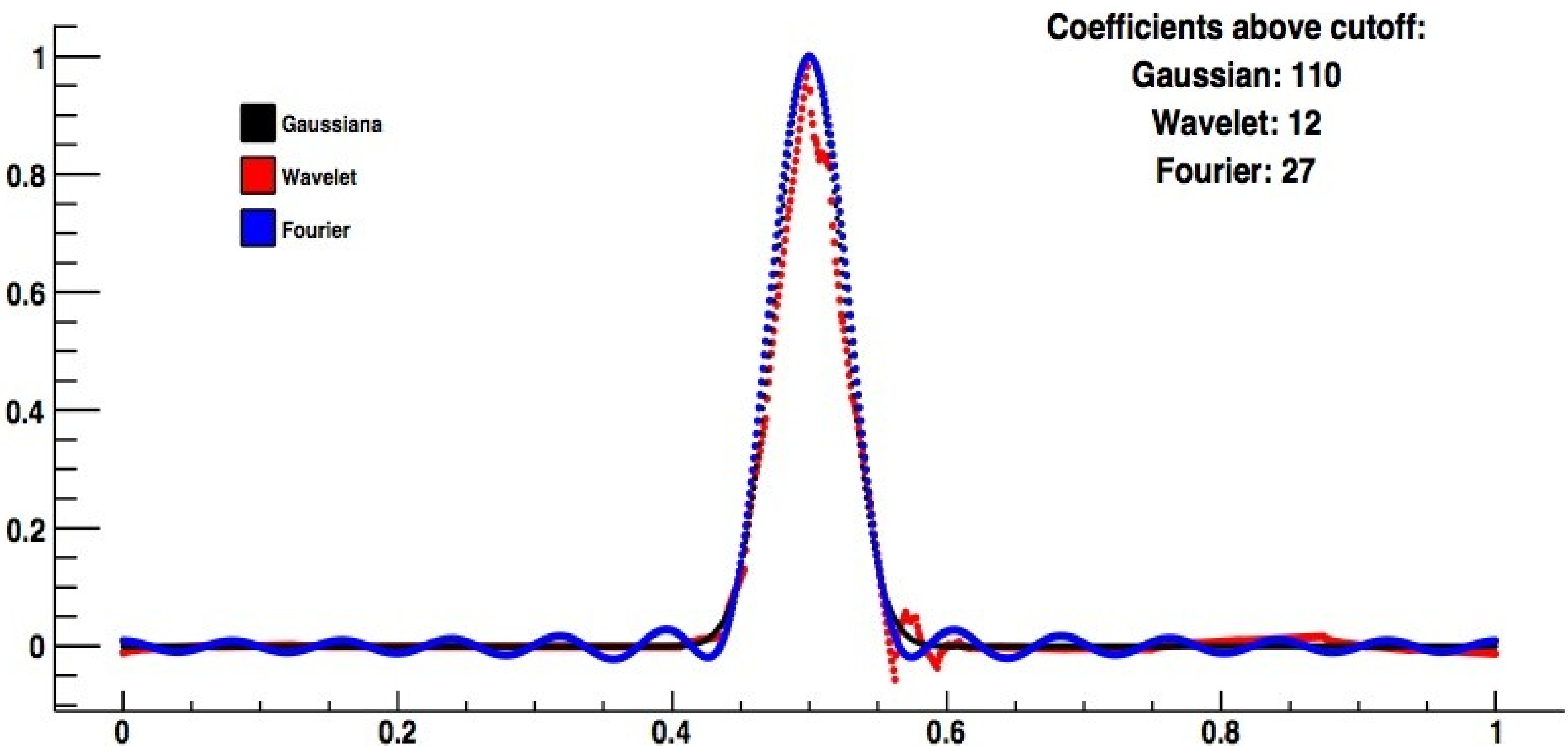
- Wavelets defined on the plane are useful for image analysis.
- Wavelets defined on the sphere are useful for spherically-distributed data
- For small angles we do not need to use wavelets on the sphere, since plane wavelets are already a good approximation.





Wavelets

- Reconstruction of signals. Original signal: gaussian.



Continuous Wavelet Transform (CWT)

- The Continuous Wavelet Transform (CWT) of a given function f(t,u) in 2 dimensions can be written as

$$\Phi(s,\tau_1,\tau_2) = \int\int f(t,u)\Psi^*_{s,\tau_1,\tau_2}(t,u)dtdu,$$

where the function f(t,u) is decomposed onto a basis of functions $\Psi_{s,\tau_1,\tau_2}(t,u)$.

- The function $\Psi_{s,\tau_1,\tau_2}(t,u)$ is obtained by means of scaling and translation of a so-called "mother-wavelet" Ψ

$$\Psi_{s,\tau_1,\tau_2}(t,u) = \frac{1}{\sqrt{s}}\Psi\left(\frac{t-\tau_1}{s},\frac{u-\tau_2}{s}\right),$$

where s is the scaling factor and τ_1 and τ_2 are the translation parameters.

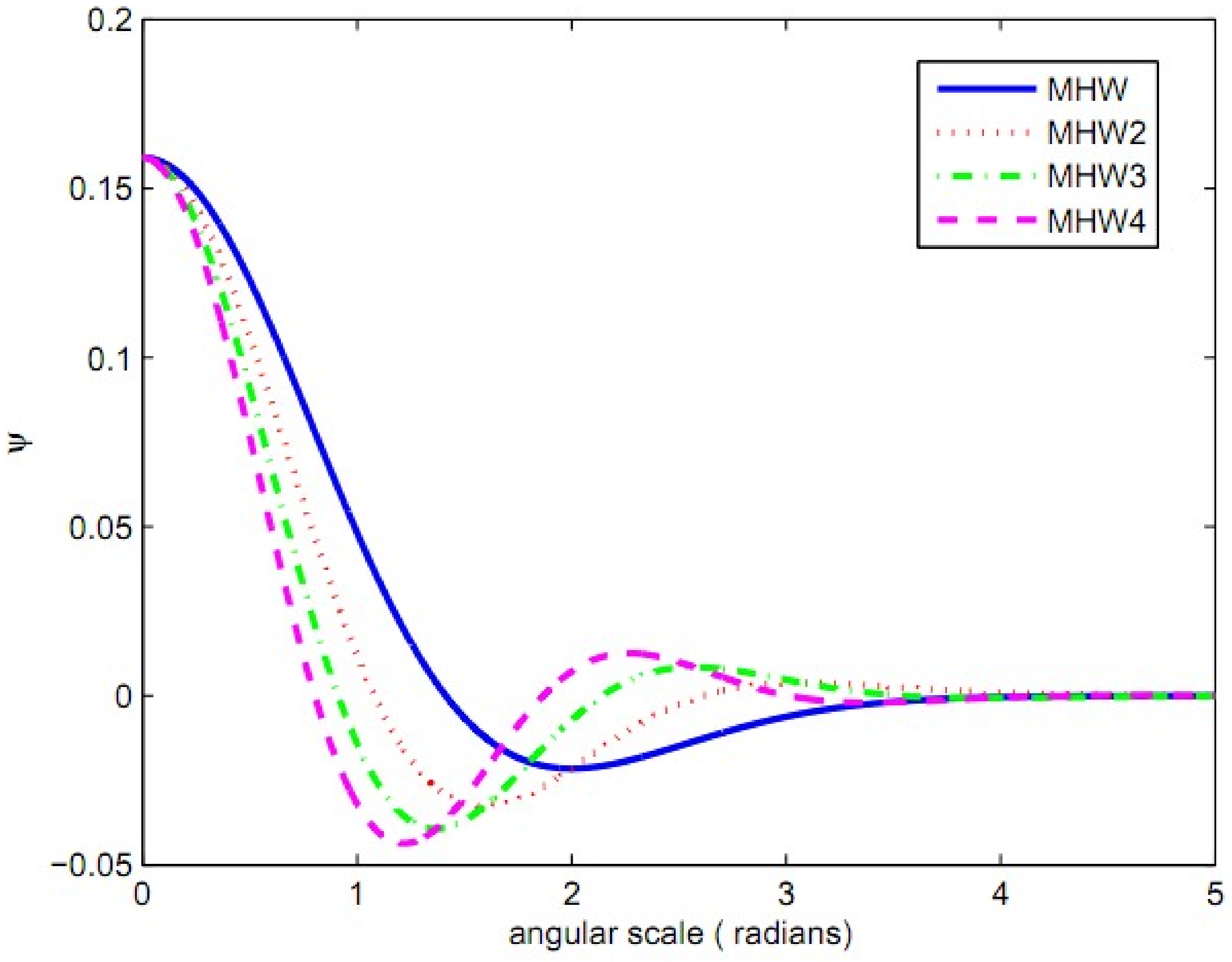
The Mexican Hat Wavelet Family (MHWF)

- The Mexican Hat Wavelet Family (MHWF) is obtained by applying the laplacian operator to the two-dimensional gaussian. A generic member of this family can be written as

$$\Psi_n(\vec{x}) = \frac{(-1)^n}{2^n n!} \nabla^{2n} \phi(\vec{x}),$$

where n is the order of the wavelet family and

$$\phi(\vec{x}) = \frac{1}{2\pi} exp\left(-\frac{\vec{x}^2}{2\sigma}\right).$$



Radial profile of MHWF1, MHWF2, MHWF3 and MHWF4. Figure extracted from Cayón et al. MNRAS, 313, 757 (2000).

arXiv:astro-ph/0604376v2

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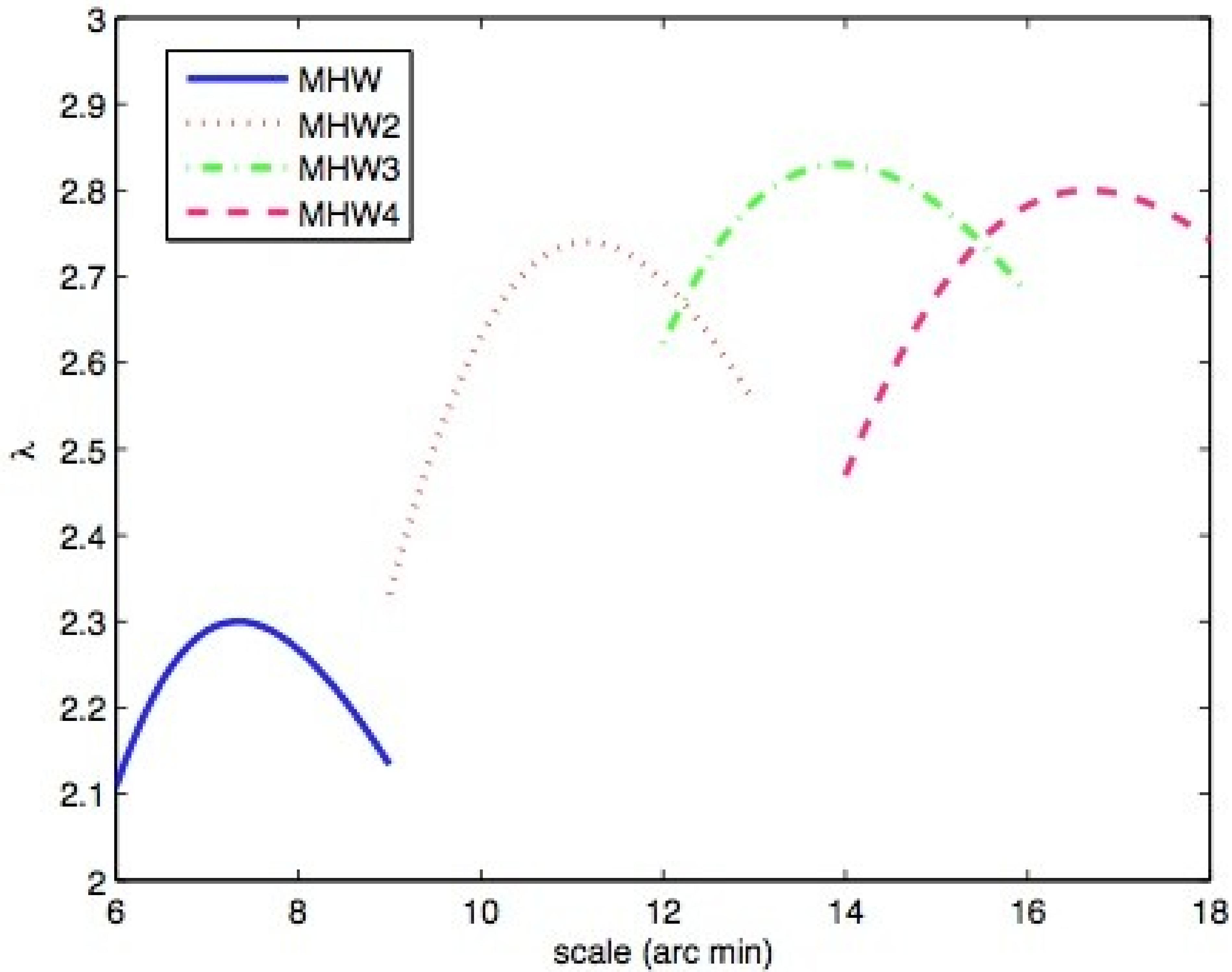


Filtering Sky Maps

- Sky maps are constructed by segmenting the celestial sphere into pixels, each one associated to a direction in a given celestial coordinate system (galactic coordinates, in our case).

- Each event detected is convolved with a probability distribution related to the angular resolution of the detector - Point Spreading Function (PSF).

- The process of convolution can be mathematically written as

$$M_f(k) = \frac{\sum_j M(j)\Phi(\vec{r_k},\vec{r_j})}{\sum_j \Phi(\vec{r_k},\vec{r_j})},$$

where $\Phi(\vec{r_k},\vec{r_j})$ is the kernel of the transformation, k and $\vec{r_k}$ are, respectively the index of the pixel being operated and its direction, $M(j)$ is the number of events in the pixel of index j in the direction $\vec{r_j}$.

Analysis Procedure

- We simulated a source at the galactic coordinates and angular size given by
  - $(l,b)=(320^\circ,30^\circ)$
  - $\sigma=2^\circ$

- Let ***A*** be the amplitude of the source with respect to the background. We have considered three cases:
  - A=10% and 100,000 events
  - A=1% and 5,000 events
  - A=0.1% and 50 events

- Four background patterns were simulated, with 500,000 events each, considering two cases:
  - full coverage with uniform acceptance
  - The Pierre Auger Observatory with an acceptance law *$\sin\theta\cos\theta$*.

- The amplification λ of the signal-to-noise ratio is calculated according to the expression

$$\lambda = \frac{w_f/\sigma_f}{w_0/\sigma_0},$$

where w is the value of the pixel in the direction of the source, σ is the root mean square (RMS) and the subscripts f and 0 refers, respectively, to the filtered and non filtered maps.

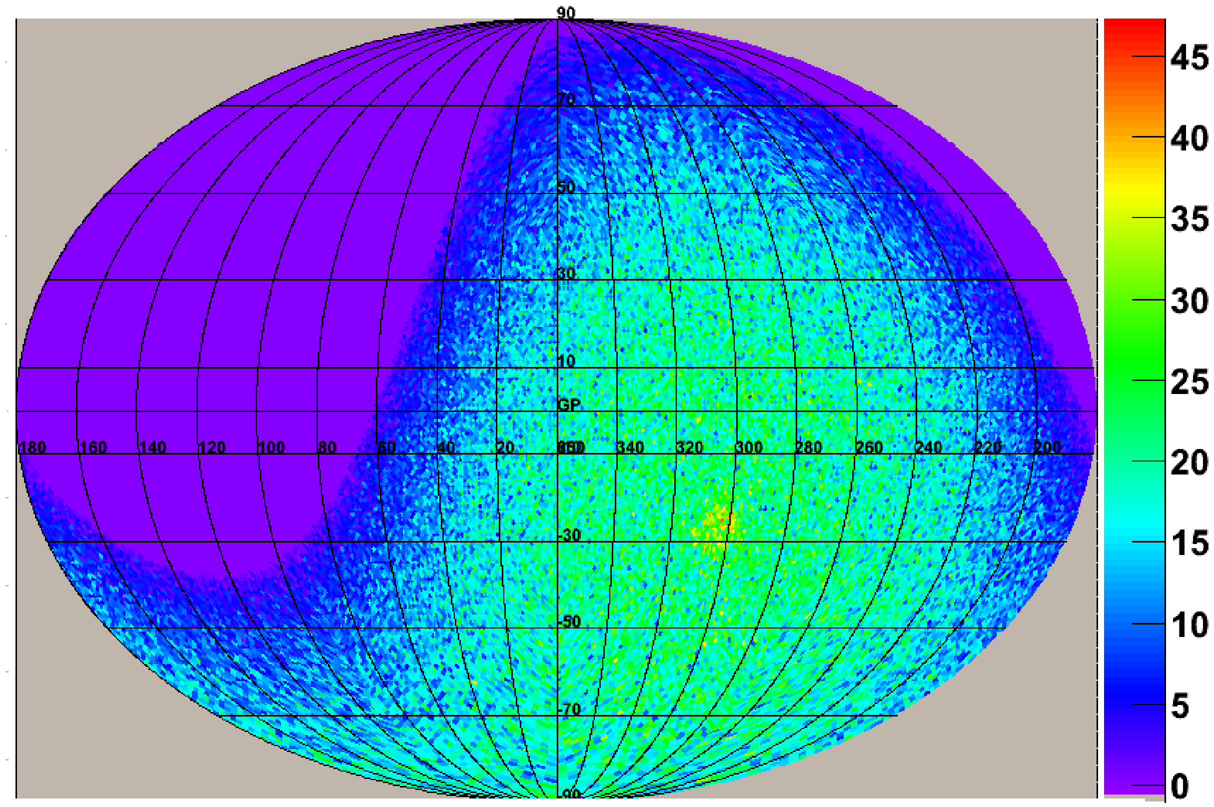
Background Patterns

- Isotropic: isotropic distribution of events.
- Dipole I: a dipole with excess in the region of the galactic center (l,b)=(0°,0°) with amplitude 7% with respect to the background.
- Dipole II: a dipole with excess in the direction (l,b)=(266.5°,-29°) with amplitude 0.5% with respect to the background.
- Sources: several sources with different angular scales σ and amplitudes A in the directions (l,b):

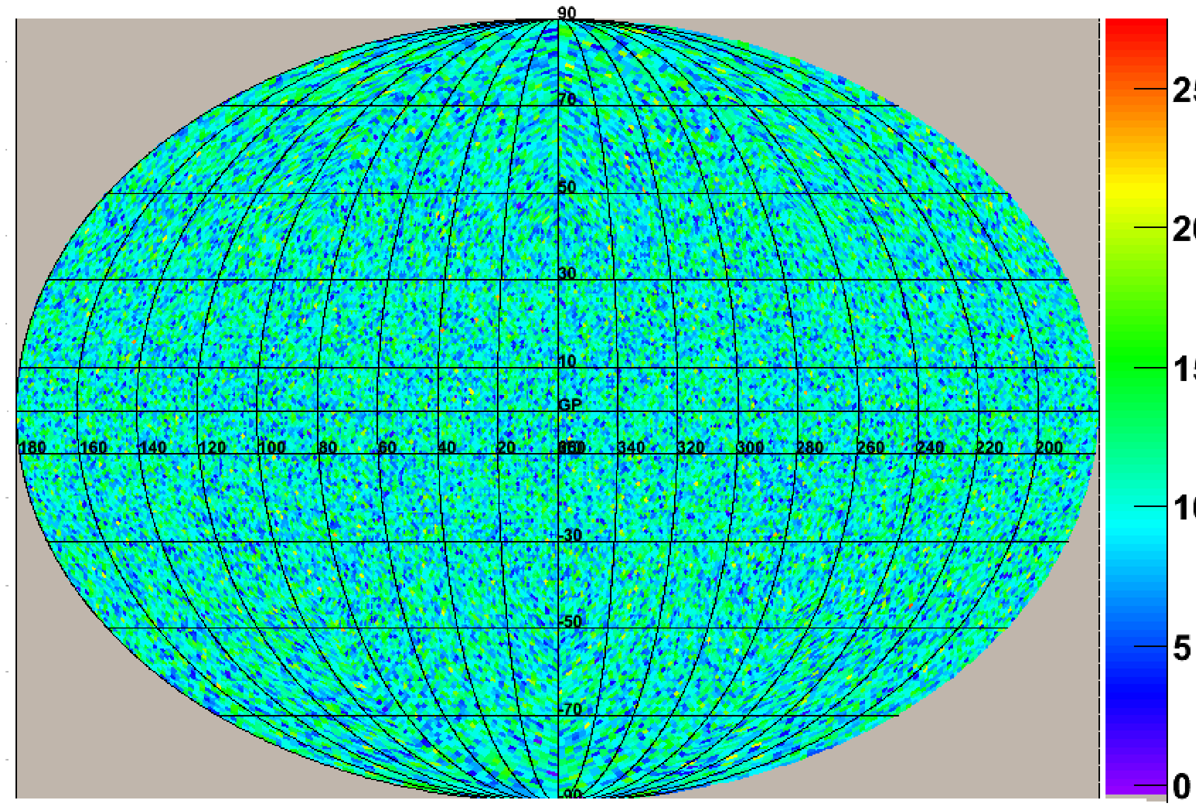
- (0°,0°) [σ = 7.0°, A = 100%]
- (320°,90°) [σ = 1.5°, A = 5%]
- (320°,−40°) [σ = 0.5°,A = 1%]
- (220°,10°) [σ = 3.0°,A = 5%]
- (100°,−70°) [σ = 2°,A = 10%]
- (240°,50°) [σ = 20°,A = 5%]
- (350°,−80°) [σ = 6.0°,A = 0.5%]
- (100°,50°) [σ = 30°,A = 50%]
- (140°,−40°) [σ = 4.0°,A = 200%]
- (60°,50°) [σ = 3.0°,A = 2%].

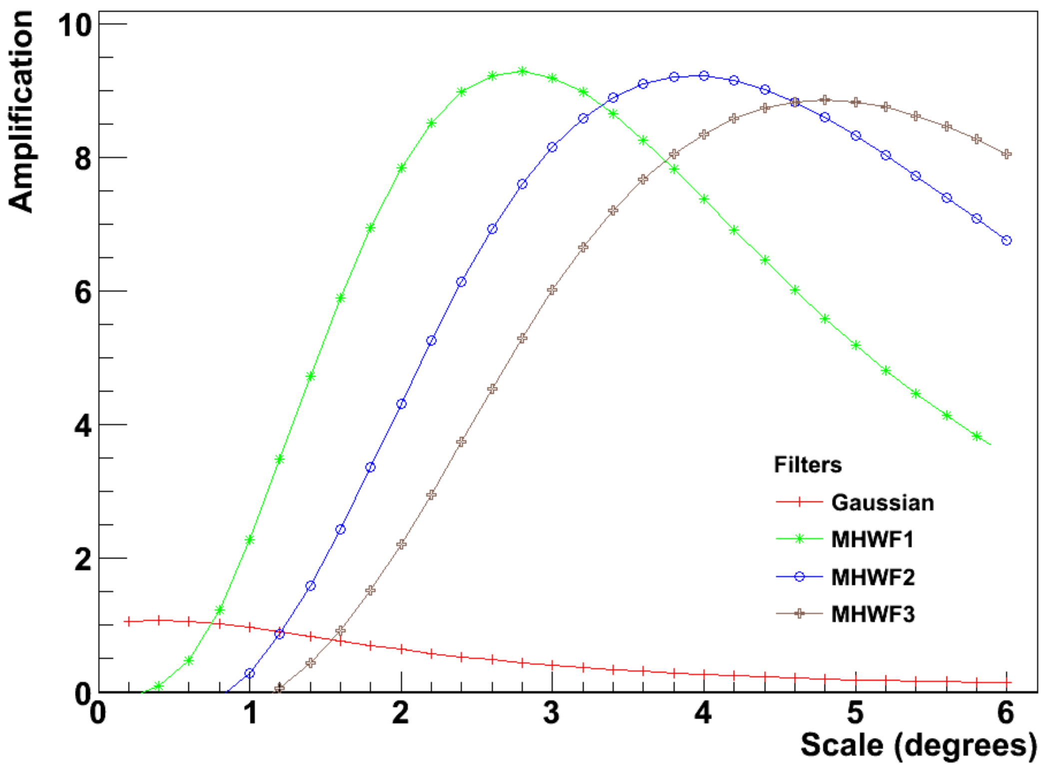
Background Patterns - Isotropic

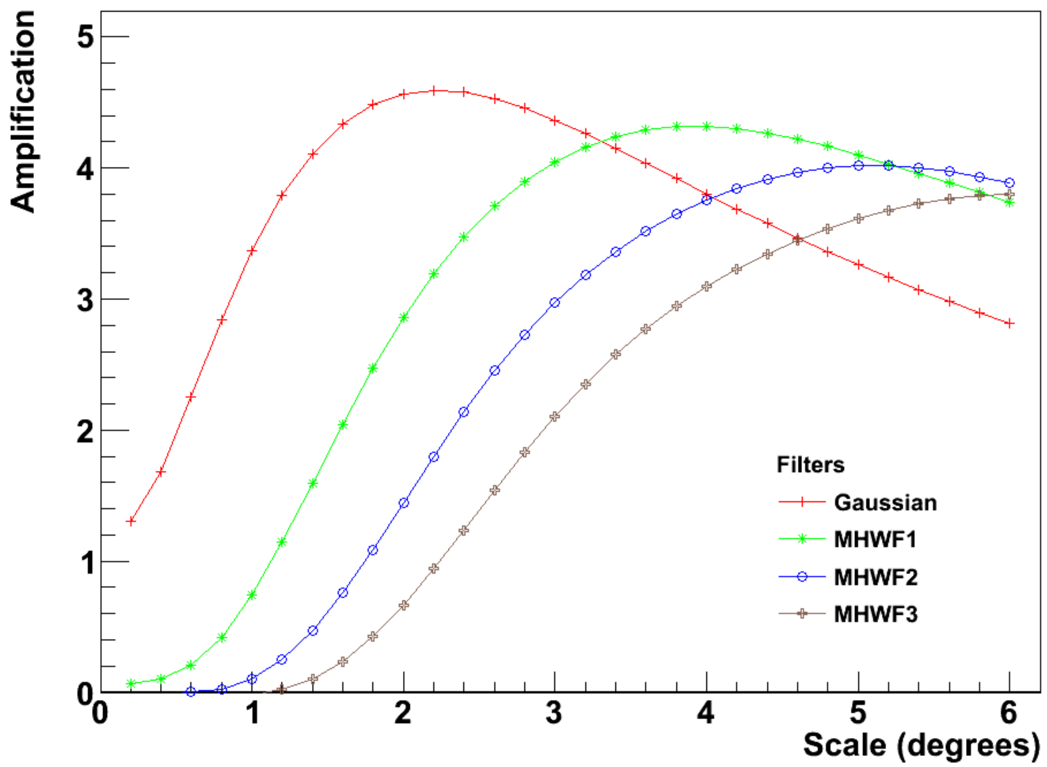
Simulated Isotropic Events Map (with acceptance law)



Simulated Isotropic Events Map (full sky)

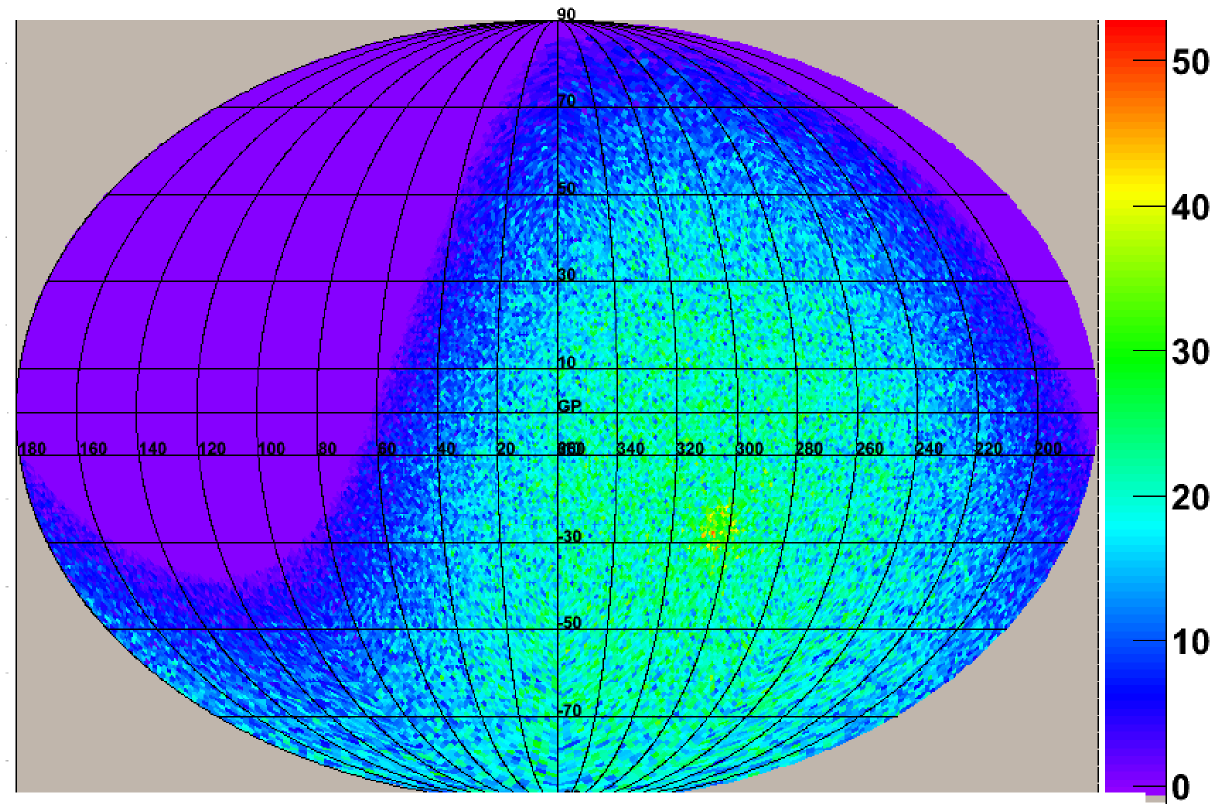




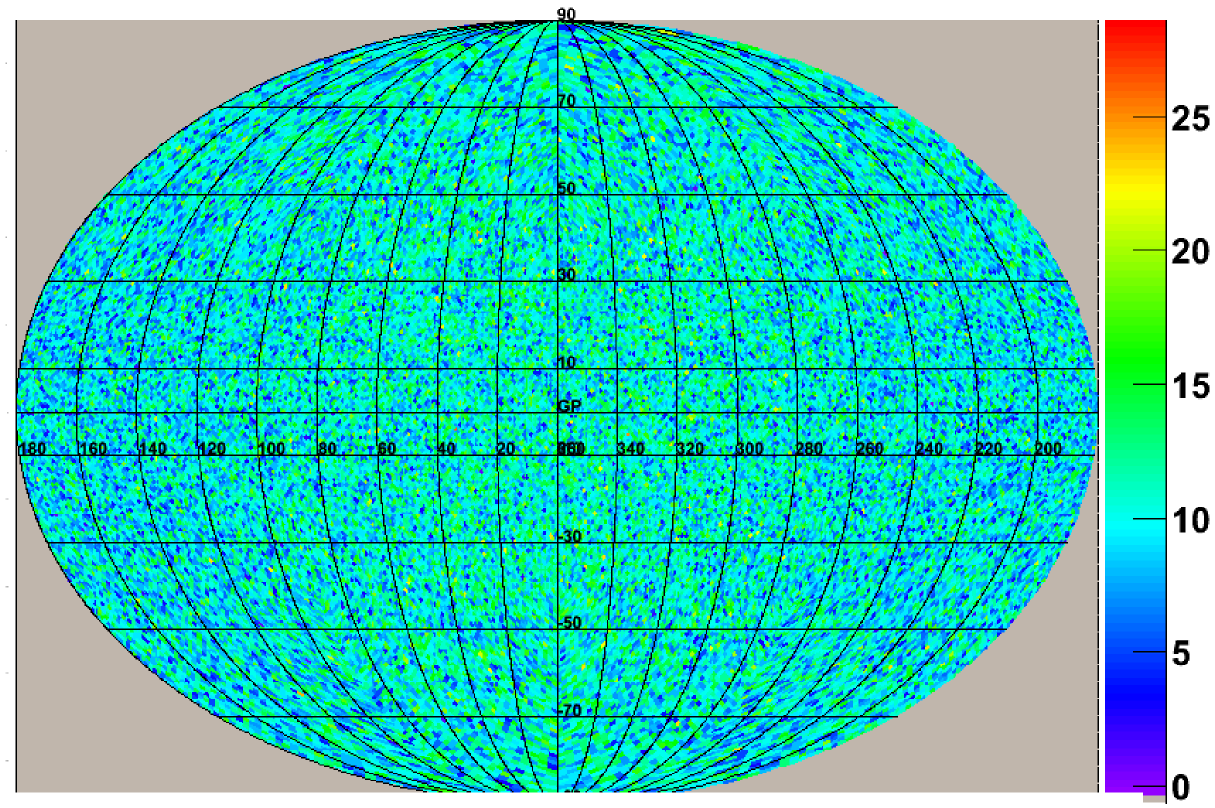


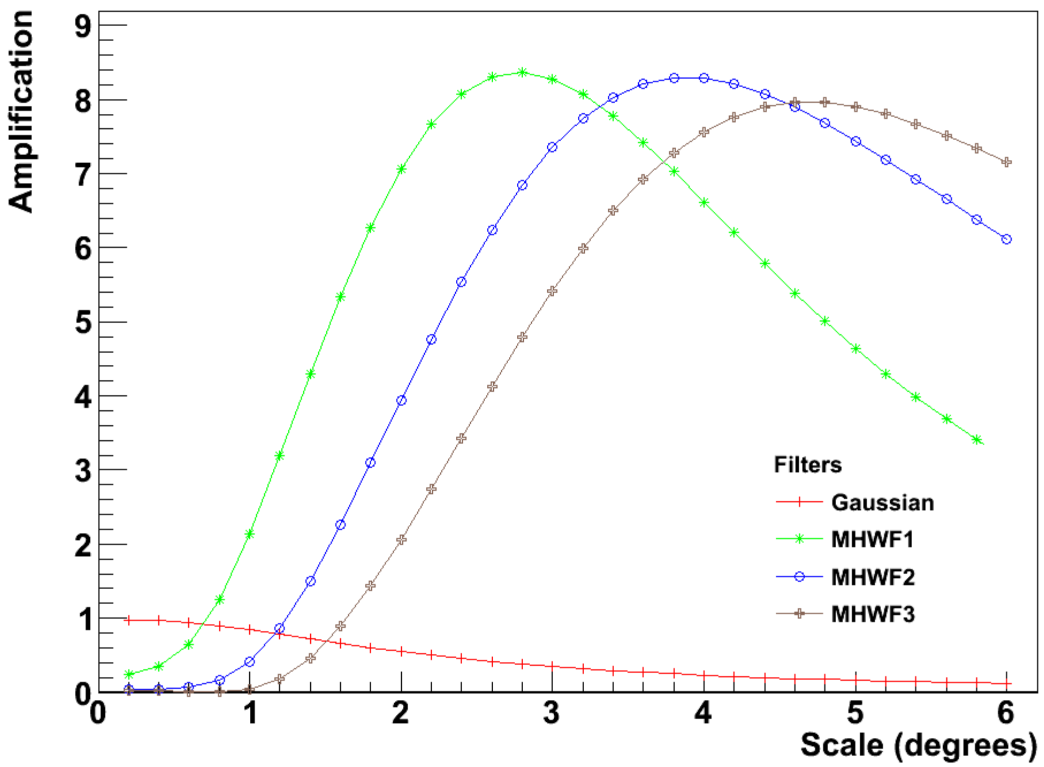
Background Patterns – Dipole I

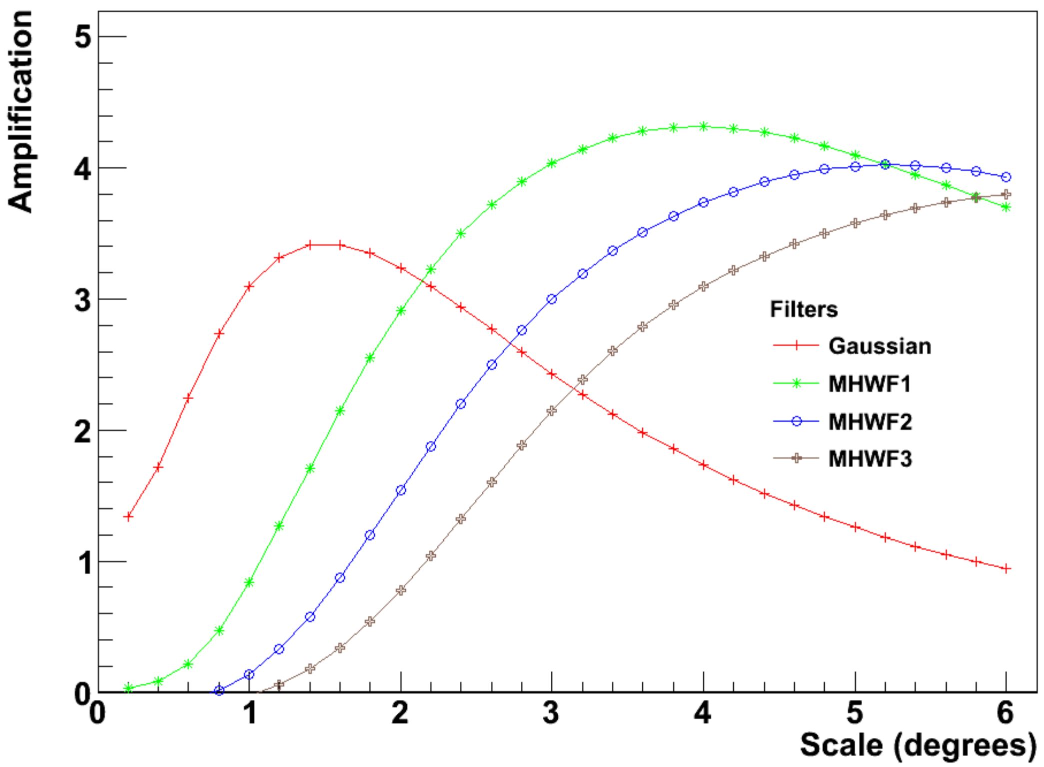
Simulated Events Map - 7% Dipole at the Galactic Center (with acceptance law)



Simulated Events Map - 7% Dipole at the Galactic Center (full sky)

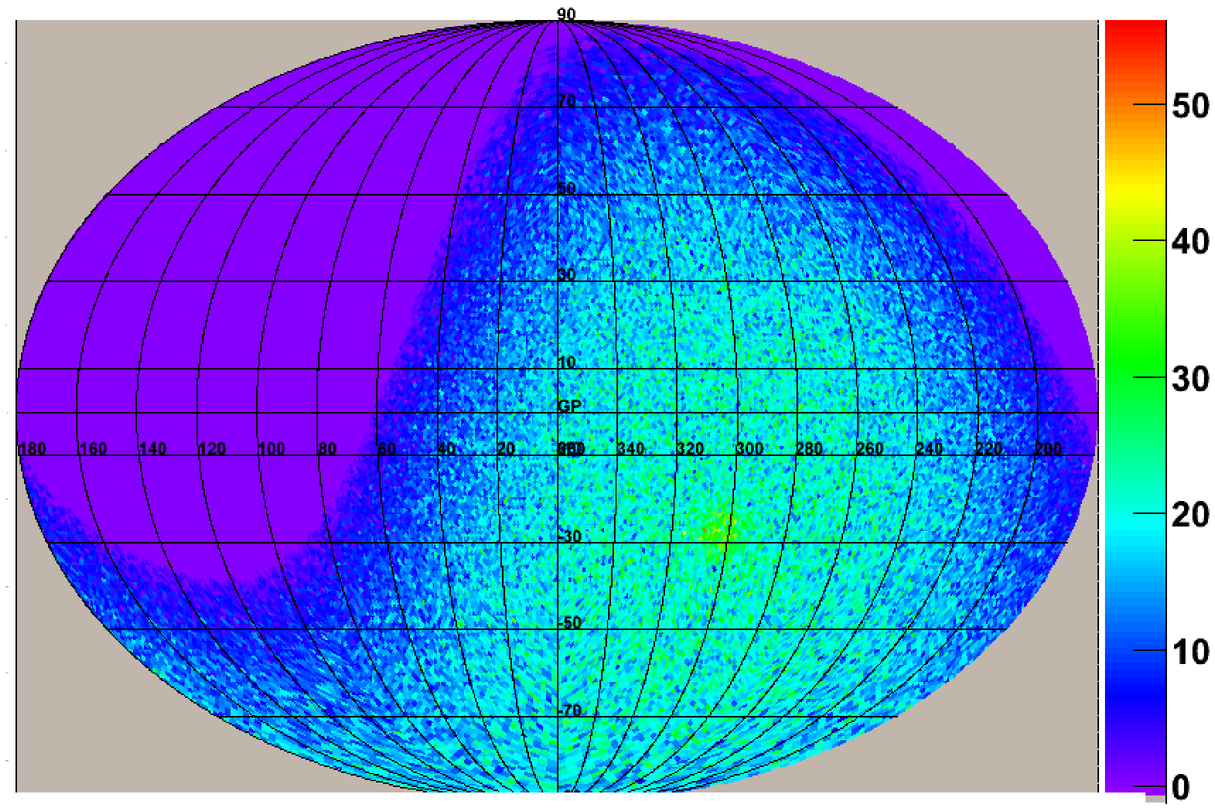




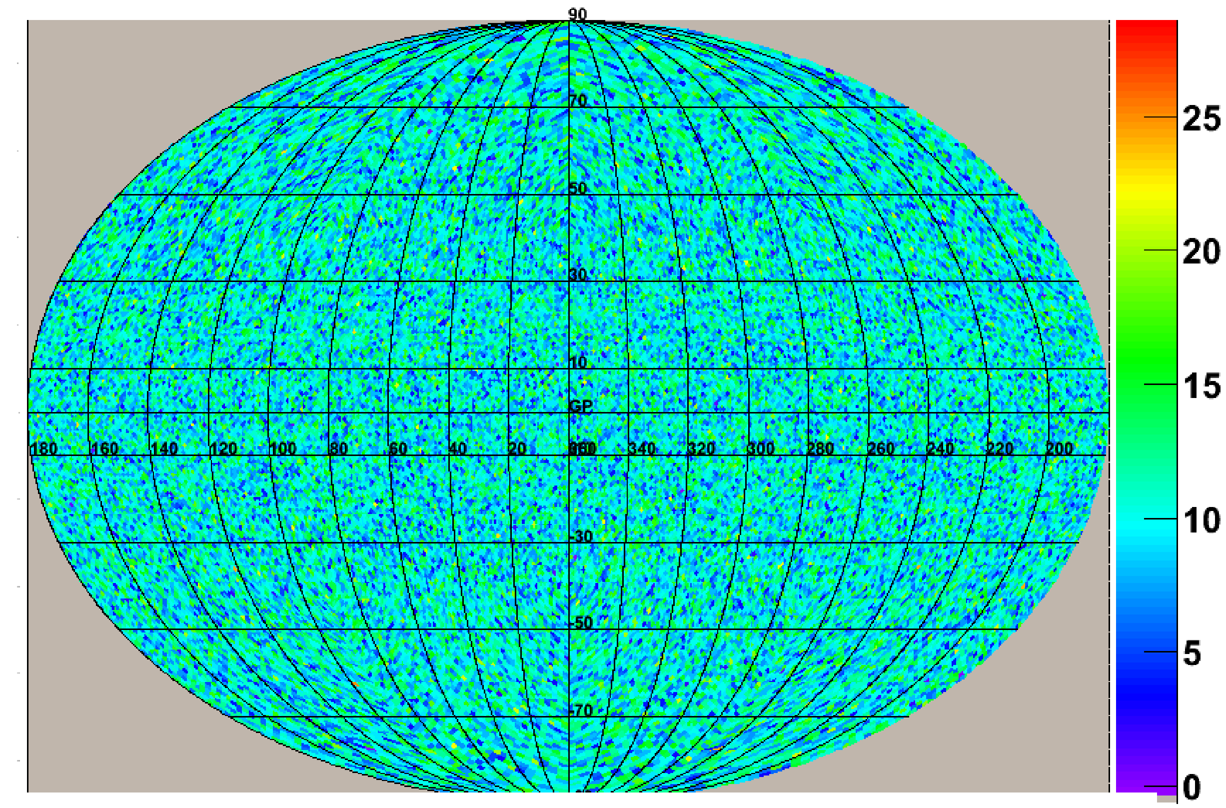


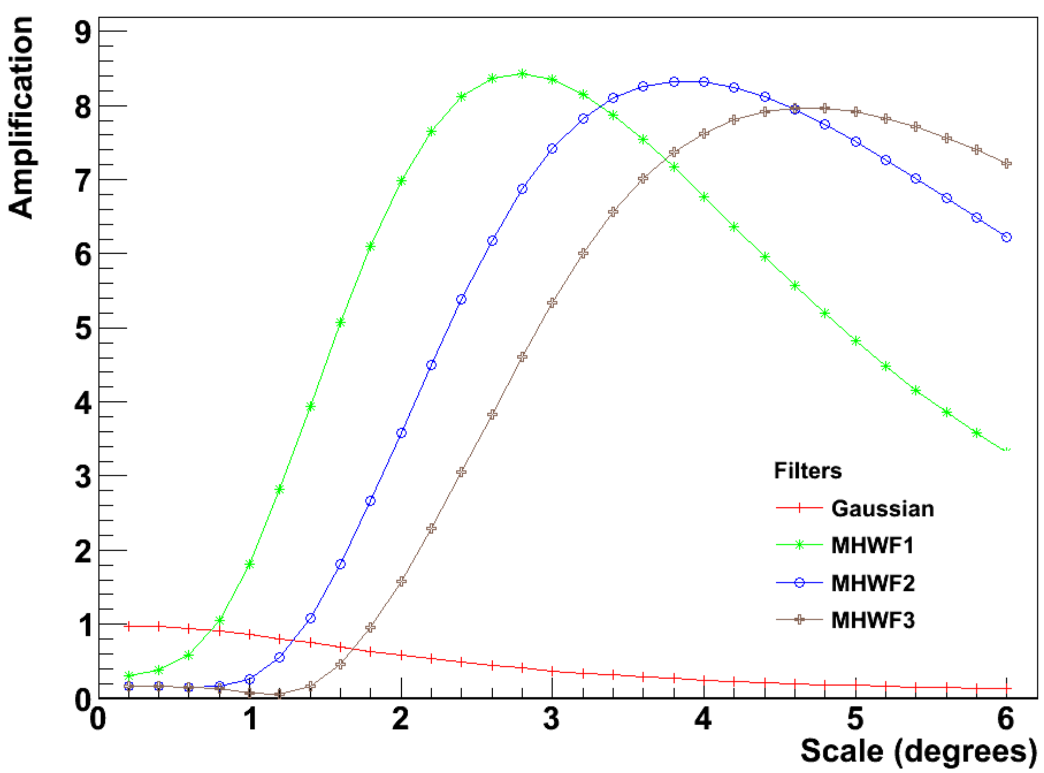
Background Patterns – Dipole II

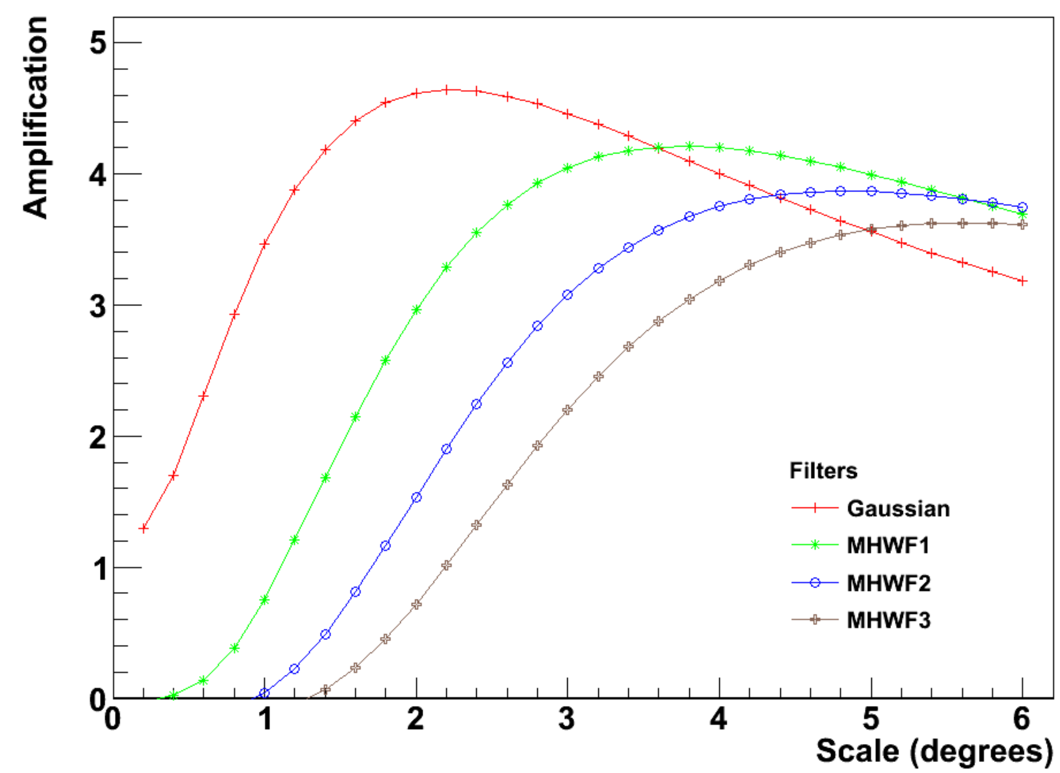
Simulated Events Map - 0.5% Dipole at (α,δ)=(0,0) (with acceptance law)



Simulated Events Map - 0.5% Dipole at (α,δ)=(0,0) (full sky)

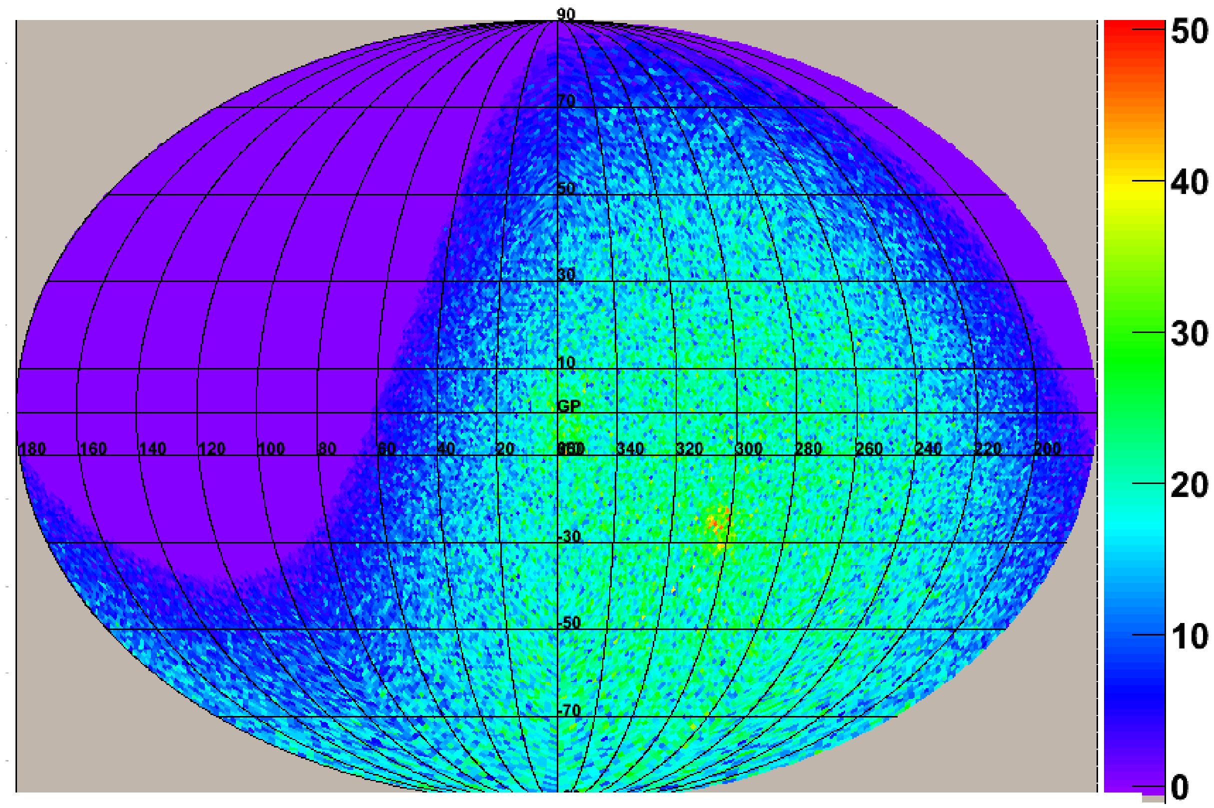




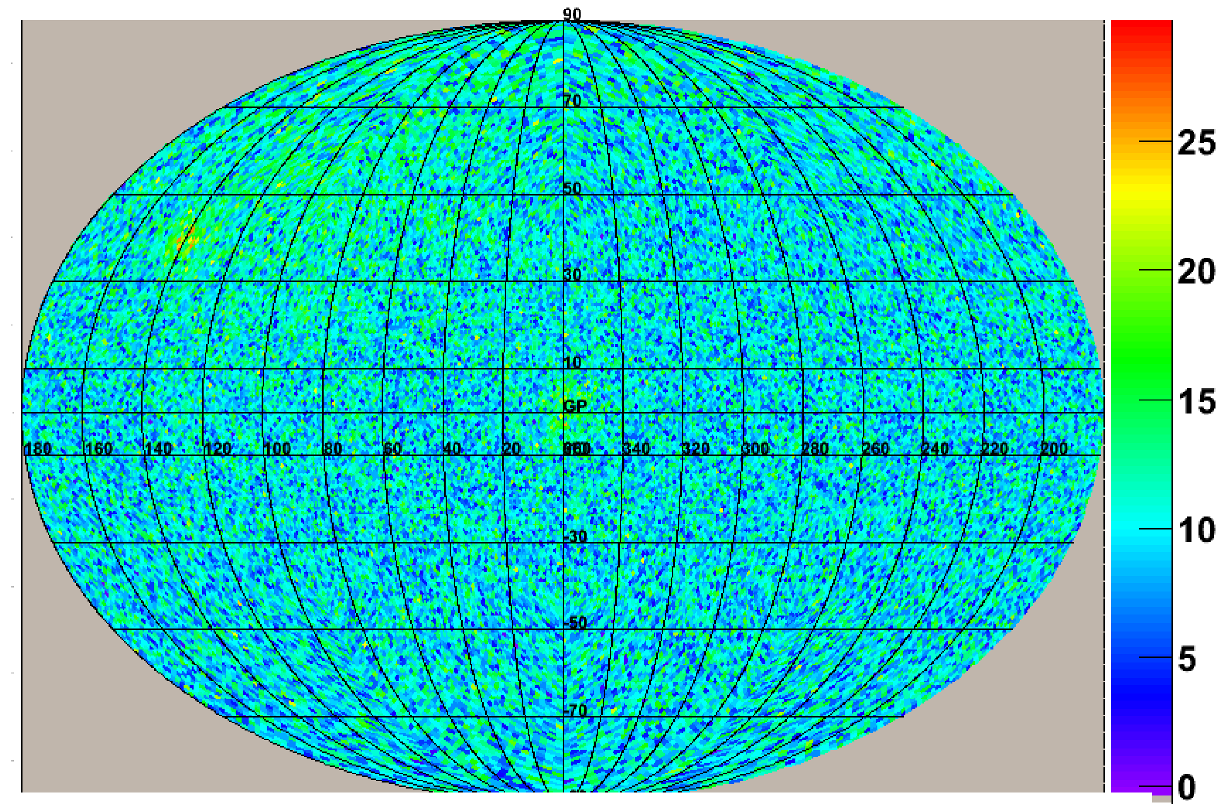


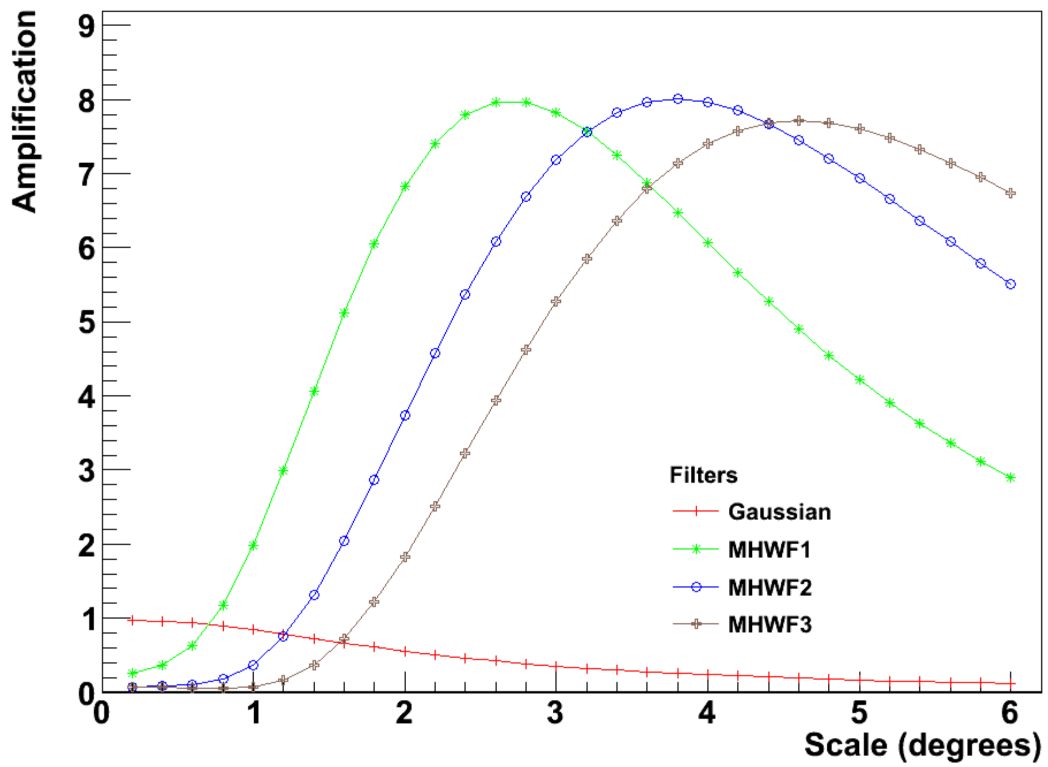
Background Patterns - Sources

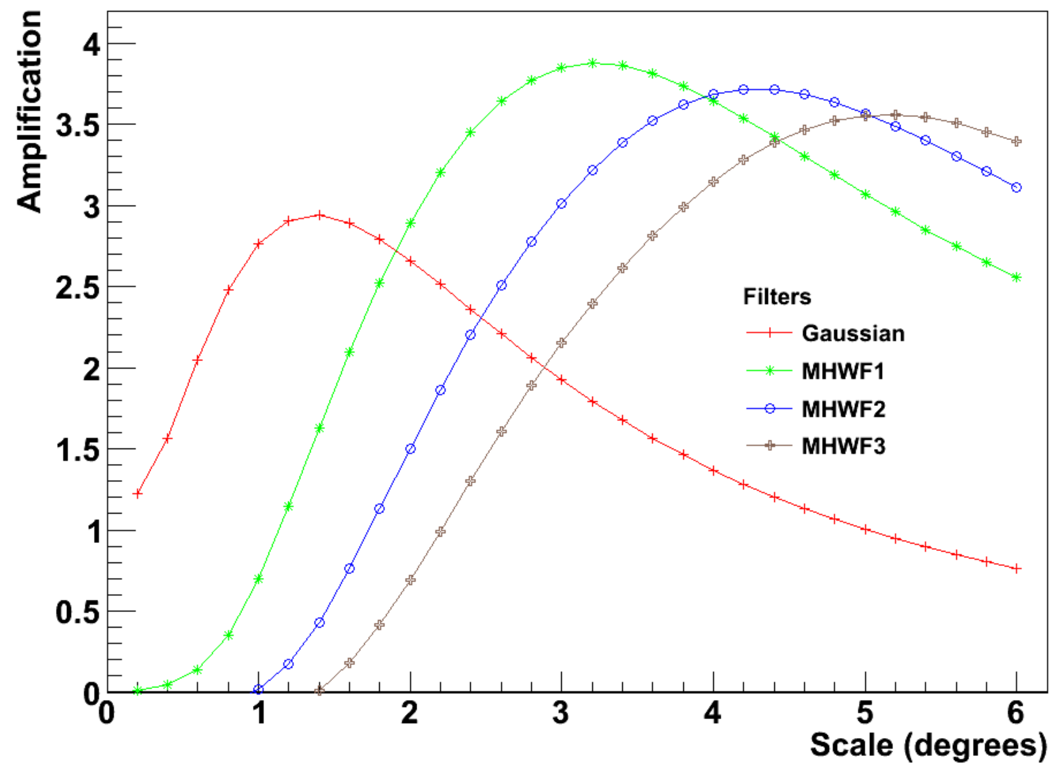
Simulated Events Map containing several sources (with acceptance law)



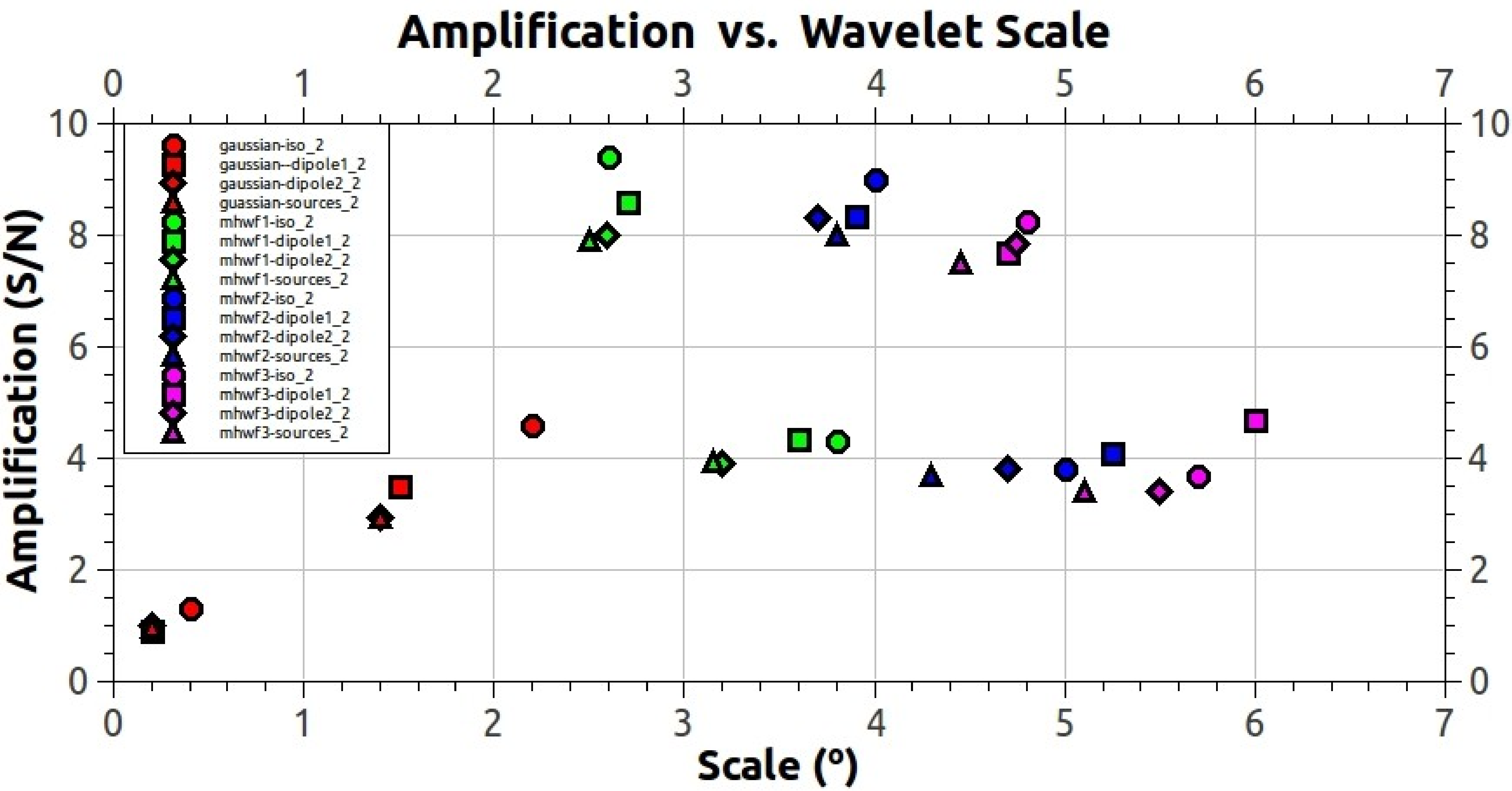
Simulated Events Map containing several sources (full sky)



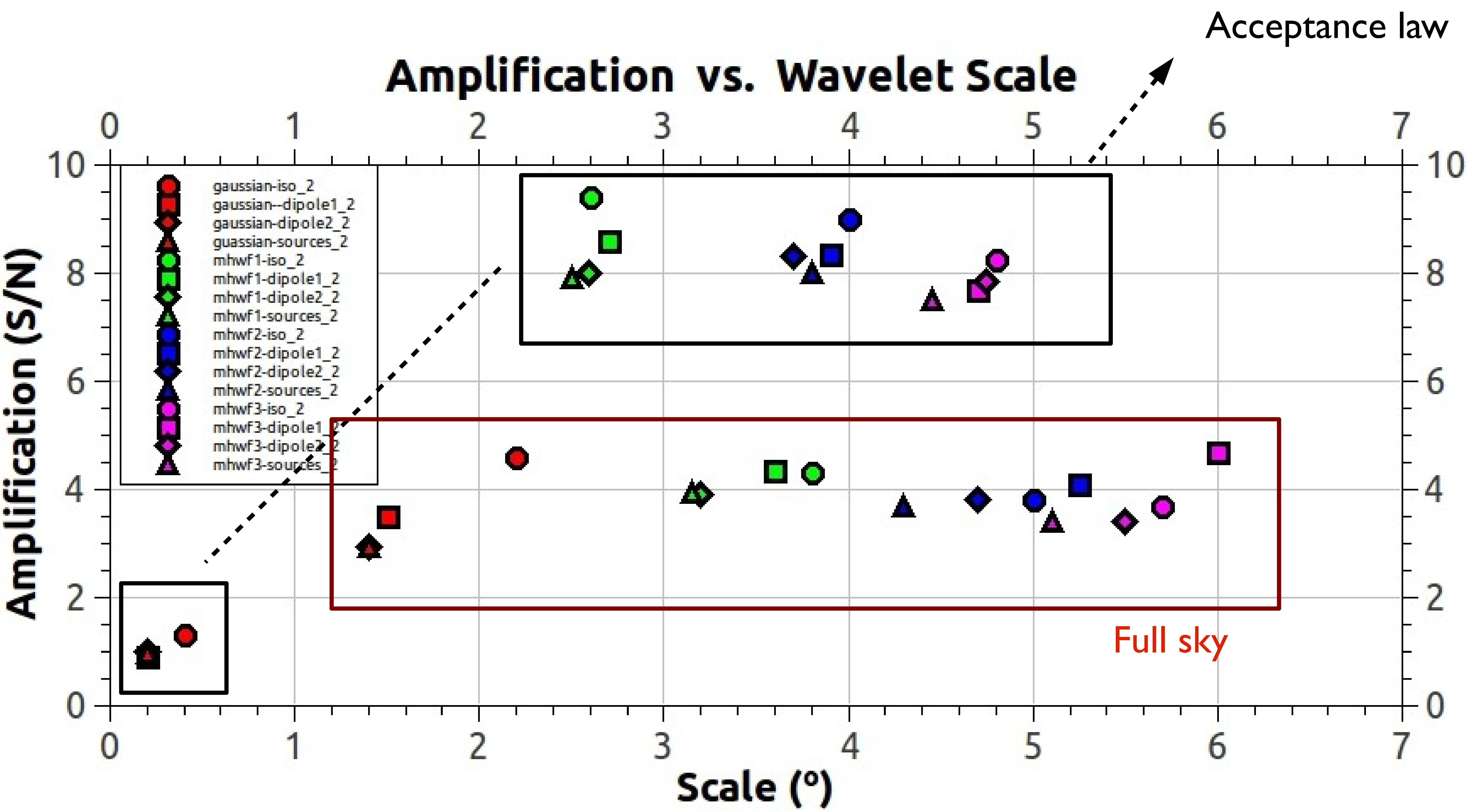




Overview of the Results



Overview of the Results



Conclusions

- We have calculated the amplification factors (S/N) for the filters (PSF):
  - gaussian, MHWF1, MHWF2 and MHWF3

    and for several background patterns
  - Isotropic, dipoles, spreading point sources

- In the case of full sky coverage with uniform acceptance, i.e., for a white noise background, the filter that maximizes the signal-to-noise ratio is the gaussian filter with dispersion (scale) equal to the angular size of the source.

- The solely existence of a non-uniform acceptance affects the power spectrum and therefore the kind of noise of the background.
  - In such a cases, the MHWF filters may provide better amplification factors.
    - Nevertheless, a kind of systematic effect of a mismatch between the optimal scale of the PSF and the angular size of the source has been observed, implying on a delocalization of the source position
    - For analysis involving the whole sky (i.e. "Blind Searches") the MHWF filters can decrease the angular resolution.

- For "window-like analysis", the gaussian filter may provide a better amplification, specially if the power spectrum within this region can be assumed uniform.

To Do List

➡ More studies on the kind of sistematic decrease of angular resolution

➡Try other filters such as Matched-Filter (MF).

➡Apply on real data on “blind searches”

Acknowledgments

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Thanks!!!